

Roll No. _____

B. Sc (Non Medical) – 5th SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Real Analysis – 9010501

TIME: 03: 00 Hrs.

Max. Marks: 40

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter;
3. Question No. 1 is Compulsory. Attempt **Five** Questions in all.
4. Students are required to attempt at least **ONE** Question from each Section. All questions carry equal marks.
5. Draw diagram whenever required.

- Q.1** a) Define Subset and Metric space. (08)
b) State Baire's category theorem.
c) Discuss Dirichlet's tests.
d) Define Continuous and monotonic function.

SECTION – A

- Q.2** Show that a constant function k is integrable and $\int_a^b k dx = k(b-a)$. (08)

- Q.3** A function f is defined on $[0, 1]$ by $f(x) = \frac{1}{n}$ for $\frac{1}{n} \geq x > \frac{1}{n+1}$, $n=1,2,3,\dots$. (08)

- Q.4** Show that the function f defined by $f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is differentiable at $x=0$ but $\lim_{x \rightarrow 0} f'(x) \neq f'(0)$. (08)

SECTION – B

- Q.5** Test the convergence of the integral $\int_1^2 \frac{dx}{\sqrt{x^4 - 1}}$. (08)

- Q.6** Let $X = R^n$ denote the set of all ordered n -tuples of real numbers for a fixed $n \in N$. (08)

- Q.7** Let $X = [0, 1]$. Consider the usual metric $d(x, y) = |x - y|$ defined for the set $X = [0, 1]$ (08)

SECTION – C

- Q.8** State and prove Bolzano-Weierstrass Theorem. (08)

- Q.9** If (X, d) is a discrete metric space and (Y, ρ) is any metric space, then show that every function $f: X \rightarrow Y$ is continuous on X . (08)

Roll No. _____

**B. Sc (Non Medical) – 5th SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Groups & Rings – 9010502**

TIME: 03: 00 Hrs.

Max. Marks: 40

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter.
3. Question No. 1 is Compulsory. Attempt **Five** Questions in all.
4. Students are required to attempt at least **ONE Question** from each **Section**. All questions carry equal marks.
5. Draw diagram whenever required.

Q.1 Attempt all the following questions: (08)

- a) Show that the set $G = \{1, 2, 3, 4, 5\}$ is not a group w.r.t. multiplication modulo 6.
- b) Define a Quotient group.
- c) Find the inverse of cycles: i) $(1\ 2\ 3\ 4)$ ii) $(2\ 3\ 4\ 5\ 6)$.
- d) Define a subring.
- e) Show that every field is a Euclidean ring.

SECTION – A

Q.2 Prove that in an abelian group $(G, .)$, $(a.b)^n = a^n . b^n$ for all integers n and all $a, b \in G$. (08)

Q.3 Prove that every subgroup of a cyclic group is cyclic. (08)

Q.4 State and prove Cayley's Theorem. (08)

SECTION – B

Q.5 State and prove The Fundamental Theorem of group homomorphism. (08)

Q.6 Prove that a finite non-zero integral domain is a field. (08)

Q.7 Define Principal Ideal Domain and show that ring of integers is Principal Ideal Domain. (08)

SECTION – C

Q.8 Find all units of $Z[\sqrt{-5}]$. (08)

Q.9 Every principal ideal domain is a unique factorization domain. (08)