

**B. Sc (Non Medical) – 6th SEMESTER
SPECIAL EXAMINATION, AUGUST-2018
Linear Algebra – 9010601**

TIME: 03: 00 Hrs.

Max. Marks: 40

INSTRUCTIONS:

1. Write Roll No. on the Question Paper.
2. Candidate should ensure that they have been provided with the correct question paper. Complaints in this regard, if any, should be made within 15 minutes of the commencement of the exam. No complaint(s) will be entertained thereafter;
3. Question No. 1 is Compulsory. Attempt **Five** Questions in all.
4. Students are required to attempt **ATLEAST ONE Question** from each UNIT. All questions carry equal marks.
5. Draw diagram whenever required.

Q.1 Attempt all the following questions:**(2 X 4 = 8)**

- a) Let \mathbb{R} be the field of real numbers, show that the set $W = \{(x, 2y, 3z) : x, y, z \in \mathbb{R}\}$ is a subspace of a $V_3(\mathbb{R})$.
- b) Is the vector $(2, -5, 3)$ in the subspace of \mathbb{R}^3 spanned by the vectors:
 $(1, -3, 2), (2, -4, -1), (1, -5, 7)$?
- c) If x and y are vectors in a real inner product space and if $\|x\| = \|y\|$, then show that $x - y$ and $x + y$ are orthogonal.
- d) Define homomorphism and isomorphism of a vector space.

UNIT-I**Q.2** Show that the union of two subspaces is a subspace iff one is contained in the other.**(08)****Q.3** If V is a vector space over a field F and if W is a subspace of V , then show the set
$$\frac{V}{W} = \{\alpha + W : \alpha \in V\}$$
 is a vector space over the field F with respect to the linear compositions

$$i) (\alpha + W) + (\beta + W) = (\alpha + \beta) + W \text{ for every } \alpha, \beta \in V.$$

(08)**Q.4** If a finite n -dimensional vector space $V(F)$ be the direct sum of its two subspaces W_1 and W_2 then show that $\dim.V = \dim.W_1 + \dim.W_2$.**(08)****UNIT-II****Q.5** If $U(F)$ and $V(F)$ are two vector spaces and T is a linear transformation from U into V , then show that range of T is a subspace of V .**(08)**

Q.6 Find the matrix representation of a linear map $T: \mathbf{R}^3 \rightarrow \mathbf{R}^3$ defined by $T(x,y,z)=(z, y+z, x+ y+ z)$ relative to the basis $\{ (1,0,1), (-1,2,1), (2,1,1) \}$. **(08)**

Q.7 Describe explicitly the linear transformation $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ such that $T(2,3)=(4,5)$ and $T(1,0)=(0,0)$ **(08)**

UNIT-III

Q.8 Apply Gram- Schmidt orthohonalization process to the vectors $\beta_1 = (1,0,1)$, $\beta_2 = (1,0,-1)$ and $\beta_3 = (0,3,4)$ to obtain an orthonormal basis $\{ \alpha_1, \alpha_2, \alpha_3 \}$ for \mathbf{R}^3 with standard inner product. **(08)**

Q.9 Let $V_1=(3,0,4)$, $V_2=(-1,0,7)$, $V_3=(2,9,11)$ be vectors in \mathbf{R}^3 equipped with the standard inner product. Obtain an orthogonal basis. **(08)**